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Relationships between a microscopic parameter and the stochastic equations for interface's evolution of two growth models

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Abstract. The relationship between a microscopic parameter p, that is related to the probability of choosing a mechanism of deposition, and the stochastic equation for the interface's evolution is studied for two different models. It is found that in one model, that is similar to ballistic deposition, the corresponding stochastic equation can be represented by a Kardar-Parisi-Zhang (KPZ) equation where both λ and ν depend on p in the following way: $\nu(p) = \nu p$ and $\lambda(p) = \lambda p^{3/2}$. Furthermore, in the other studied model, which is similar to random deposition with relaxation, the stochastic equation can be represented by an Edwards-Wilkinson (EW) equation where ν depends on p according to $\nu(p) = \nu p^2$. It is expected that these results will help to find a framework for the development of stochastic equations starting from microscopic details of growth models.

PACS. 68.35.Ct Interface structure and roughness – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 02.50.-r Probability theory, stochastic processes, and statistics – 81.15.Aa Theory and models of film growth

1 Introduction

The characterization of the roughness of a surface is an important issue in technology and science. Mechanical problems concerning wear, friction, or adhesion show a crucial dependence on the smoothness of the surfaces that get into contact. It is also known that surface roughness affects the electrical, magnetic and optical properties of thin films, which makes the development of better controlled surface growth techniques an important line of research. These techniques generally show growth regimes with common spatio-temporal features, as for instance the appearance of scale invariant rough surfaces [1]. Natural processes, such as the propagation of forest fires or the growth of bacterial colonies also show interfacial scale invariant behavior [1]. This scale invariance is revealed by scaling exponents and functions that has to be measured in order to classify the growth processes into universality classes.

Models of growing interfaces may be defined and studied either on discrete lattices or by means of continuous equations. Discrete models are defined by a set of rules that provide a detailed microscopic description of the evolution of the surface. Some very well known discrete models are the random deposition (RD), the random deposition with surface relaxation (RDSR), the ballistic deposition (BD), the eden growth model, etc. [1–4]. In these models the interface is described by a discrete set $h(i, t)$ that represents the height of site i at time t. The interface has L^d sites, where L is the linear size and d is the dimensionality of the substrate. The interface of the aggregate is characterized through scaling of the interface width $W(L, t) \equiv \sqrt{1/L^d \sum_{i=1}^{L^d} [h(i, t) - \langle h(t)\rangle]^2}$. For this purpose, the Family-Vicsek phenomenological scaling approach [5] has proved to be very successful for the description of the dynamic evolution of growing interfaces. In fact, it may be expected that $W(L, t)$ would show the spatio-temporal scaling behavior given by [5]: $W \propto L^{\alpha}$ for $t \gg t_c$ and $W(t) \propto t^{\beta}$ for $t \ll t_c$, where $t_c \propto L^Z$ is the crossover time between these two regimes. The scaling exponents α, β and $Z = \alpha/\beta$ are called roughness, growth and dynamic exponents, respectively.

In contrast to the microscopic details of the growing mechanisms of the interface, continuous equations focus on the macroscopic aspects of the roughness. Essentially, the aim is to follow the evolution of the coarse-grained height function $h(\mathbf{x}, t)$ using a well-established phenomenological approach that take into account all the relevant processes that survive at a coarse-grained level. This procedure normally leads to stochastic nonlinear

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partial differential equations that may be written as follows [1,6–8]

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = G_i\{h(\mathbf{x},t)\} + F + \eta(\mathbf{x},t), \qquad (1)$$

where the index i symbolically denotes different processes, $G_i\{h(\mathbf{x},t)\}$ is a local functional that contains the various surface relaxation phenomena and only depends on the spatial derivatives of $h(\mathbf{x},t)$ since the growth process is determined by the local properties of the surface. Also, F denotes the mean deposition rate and $\eta(\mathbf{x},t)$ is the deposition noise that determines the fluctuations of the incoming flux around its mean value F. It is usually assumed that the noise is spatially and temporally uncorrelated.

In order to establish the correspondence between a continuous growth equation and a discrete model one can follow at least three different methods: (i) to numerically simulate the model and compare the obtained scaling exponents with those of the corresponding continuous equation, (ii) to develop a set of plausibility arguments using physical principles and (iii) to derive the continuous equation analytically starting from a given discrete model.

There are few papers in the direction of the last method. For example, a systematic approach proposed by Vvedensky *et al.* [9], where the continuous equations can be constructed directly from the growth rules of some discrete models, based on the master equation description, has been applied successfully [9–12]. This procedure requires the regularization step, in which the non-analytic quantities are expanded and replaced with analytic quantities, *i.e.*, the step function is approximated by an analytic shifted hyperbolic tangent function expanded in a Taylor series. As pointed out by Předota and Kotrla [11], the choice of the regularization scheme for the step function is ambiguous. Thus, the coefficients in the derived continuum stochastic equation cannot be determined uniquely. Another method has shown the connection between the ballistic deposition discrete model and the Kardar-Parisi-Zhang (KPZ) equation in $d = (1 + 1)$ dimensions. However, this method is not successful in $d = (2 + 1)$ dimensions [13].

Within this context, the aim of the present work is to study the connection between parameters related to the microscopic dynamics of two discrete models, similar to RDSR and BD, with their continuous equations in at least $d = (1 + 1)$, $(2 + 1)$ and $(3 + 1)$ dimensions, respectively.

The manuscript is organized as follow: firstly, in Section 2, the models and previous results corresponding to low dimensionality simulations will be shortly reviewed. Subsequently, new results of numerical simulations performed in higher dimensions will be presented and discussed in Section 3. In order to study the connection between the parameters related to the microscopic dynamics and their corresponding continuous equations, a phenomenological stochastic growth equation will be proposed in Section 4. Finally, our conclusions will be stated in Section 5.

2 Description of the models and previous results

In the first discrete growth model, namely the RDSR/RD model, particles of a single kind are aggregated according to the rules of random deposition with surface relaxation (RDSR) with probability p and according to the rules of random deposition (RD) with probability $(1 - p)$ [14]. In the second discrete growth model, namely the BD/RD model, particles are aggregated according to the rules of ballistic deposition (BD) with probability p and according to the rules of random deposition (RD) with probability $(1 - p)$ [15].

It should be noticed that the study of this kind of competitive growth models has recently attracted growing attention [16–21]. This interest is motivated by the fact that in nature, and also in laboratories, actual growth processes are most likely due to the interplay of various competing mechanisms that may operate at different spatio-temporal scales.

In the RD growth model a column is randomly chosen along the width of the sample of side L. Then a particle is allowed to fall vertically until it reaches the top of the selected column, whereupon it is deposited. For this model, $W(t)$ does not saturate due the lack of lateral correlations, so $W(t) \propto t^{\beta_{\rm RD}}$ is independent of L with $\beta_{\rm RD} = \frac{1}{2}$.

In the lattice version of the RDSR growth model a particle is released from a random position above the surface and falls vertically until it reaches the top of the selected column (just as in the case of the random deposition model). Then, the deposited particle is allowed to relax to a nearest neighbor column if the height of the neighboring column is lower than the one corresponding to the selected column.

It is well known that the RDSR model can be described by the Edwards-Wilkinson equation which has the form [1,6,7]:

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = F + \nu_o \nabla^2 h(\mathbf{x},t) + \eta(\mathbf{x},t), \qquad (2)$$

where ν_o plays the role of an effective surface tension, since the $\nu_o \nabla^2 h(\mathbf{x},t)$ term tends to smooth the interface. Equation (2) can be solved in Fourier space and the following values of the exponents are obtained: $z = 2$, $\alpha = (2-d)/2$ and $\beta = (2 - d)/4$. This model has an upper critical dimension $d_c = 2$ above which one has $\alpha = 0$ and $\beta = 0$

The lattice version of BD is rather simple to describe: particles fall vertically onto the substrate from a random position above the surface of side L. When a particle reaches the surface, it sticks on the first site encountered that is a nearest-neighbor of an already deposited particle. Due to this constrain the growth of an interface essentially parallel to the substrate is observed. This model can be described by the Kardar-Parisi-Zhang (KPZ) equation [1,8]

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = F + \nu_o \nabla^2 h(\mathbf{x},t) + \frac{\lambda}{2}[\nabla h(\mathbf{x},t)]^2 + \eta(\mathbf{x},t). \qquad (3)$$

In this equation the nonlinear term represents the lateral growth or the appearance of a driven force. In $d = (1 + 1)$ dimensions, the exponents of this equation are $z = 3/2$, $\alpha = 1/2$ and $\beta = 1/3$. In $d = (2 + 1)$ dimensions, the analytical solution is not known and numerical simulations indicate $\alpha \sim 0.40$ and $\beta \sim 0.24$ [22].

For both the RDSR/RD and the BD/RD models in $d = (1 + 1)$ dimensions, the saturation value W_s sensitively depends on p: saturation takes place at longer times for smaller values of p, while the width of the interface is smaller for larger p-values. Furthermore, in both models, three different regimes and the corresponding crossovers can easily be observed. For short times, say $t < t_{x1}$, the random growth of the interface is observed (the RD process dominates). At this stage, correlations have not been developed yet and $W(t) \propto t^{\beta_{\rm RD}}$ $(t < t_{x1})$ holds. During an intermediate time regime, say $t_{x1} < t < t_{x2}$, correlations develop since the RDSR (BD) process now dominates leading to $W(t) \propto t^{\beta_{\rm RDSR}}$ $(W(t) \propto t^{\beta_{\rm BD}})$. At a later stage, for $t > t_{x2}$, correlations can not longer grow due to the geometrical constraint of the lattice size and saturation is observed. The interface width $W_s(L, p)$ and the characteristic crossover time t_{x2} behaves as [14,15]:

$$W_s(L, p) \propto L^{\alpha_X} \, p^{-\delta} \qquad (p > 0), \tag{4}$$

and

$$t_{x2}(L, p) \propto L^{Z_X} \, p^{-y} \qquad (p > 0), \tag{5}$$

where δ and y are exponents and $X \equiv$ RDSR, BD depending on the model. For $t_{x1} < t < t_{x2}$ one has

$$W(t, p) \propto t^{\beta_X} \, p^{-\gamma} \qquad , \tag{6}$$

where γ is also a characteristic exponent.

The exponents γ, δ and y are not independent. Using a phenomenological dynamic scaling ansatz it can be shown that the following relationship between exponents holds at least in in $d = (1 + 1)$ dimensions [14,15]:

$$y\beta_X - \delta + \gamma = 0. \tag{7}$$

For the RDSR/RD model in $d = (1 + 1)$ dimensions, one has $y \equiv 2 \quad (1.97 \pm 0.05)$, $\delta \equiv 1 (0.97 \pm 0.04)$, and $\gamma \equiv \frac{1}{2} \quad (0.51 \pm 0.05)$. Also, for the BD/RD model $y \equiv 1 \quad (0.97 \pm 0.02)$, $\delta \equiv \frac{1}{2} \quad (0.45 \pm 0.01)$, and $\gamma \equiv \frac{1}{6} \quad (0.17 \pm 0.01)$. Notice that the rational numbers are the conjectured exact values for the new exponents, while the values between brackets are the numerical estimations [14,15].

3 Numerical simulations in higher dimensions

For both models, numerical Monte Carlo simulations were performed in $(2+1)$ and $(3+1)$ dimensions using lattices of side L, with $16 \leq L \leq 256$, and periodic boundary conditions in the direction perpendicular to the growing surface. As usual, a Monte Carlo time step (mcs) involves the deposition of L^d particles. Depending on L and d, the results are averaged over 10^2–10^3 different runs.

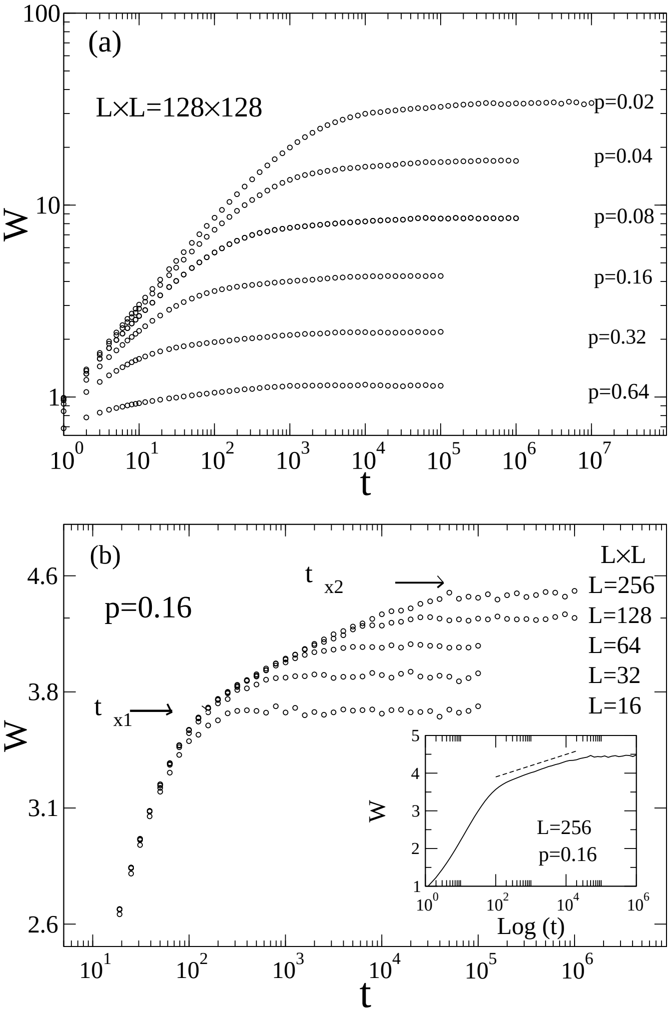

Fig. 1. Log-log plots of the interface width (W) *versus* time for the RDSR/RD model in $d = (1 + 2)$ dimensions obtained for: (a) $L = 128$ and different values of p as indicated in the figure, and (b) $p = 0.16$ and lattices of different size, as indicated in the figure. In Figure (b), the location of t_{x1} and t_{x2} is indicated by arrows for the data corresponding to $L = 256$. The inset of Figure (b) shows a linal-logarithmic plot of W *versus* t where the RDSR dominated regime with $\beta = 0$ is shown by means of a dashed line. More details in the text.

Figure 1a shows log-log plots of W *versus* t obtained for the RDSR/RD model in $d = (2 + 1)$ dimensions taking $L = 128$ and different values of p. Also, Figure 1b shows plots of W *versus* t for lattices of different size but keeping $p = 0.16$ constant. For this model, as in $d = (1 + 1)$ dimensions [14], three different regimes can easily be observed. For short times, say $t < t_{x1}$, the random growth of the interface is observed (the RD process dominates). So, $W(t) \propto t^{\beta_{\rm RD}}$ with $\beta_{\rm RD} = 1/2$. During an intermediate time regime, say $t_{x1} < t < t_{x2}$, correlations develop since the RDSR process now dominates leading to $W(t) \propto t^{\beta_{\rm RDSR}}$ with $\beta_{\rm RDSR} = 0$ (this exponent is the exact one for the RDSR model in $d = (1 + 2)$

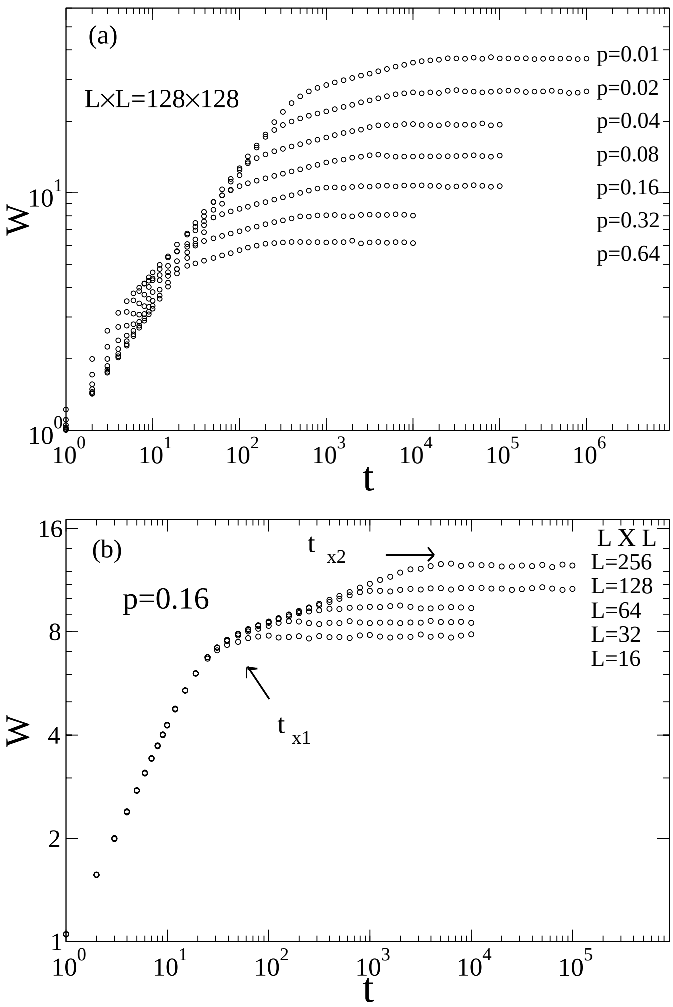

Fig. 2. Log-log plots of the interface width (W) *versus* time for the BD/RD model in $d = (2+1)$ dimensions obtained for: (a) $L = 128$ and different values of p as indicated in the figure, and (b) $p = 0.16$ and lattices of different size, as indicated in the figure. In Figure (b), the location of t_{x1} and t_{x2} is indicated by arrows for the data corresponding to $L = 256$. More details in the text.

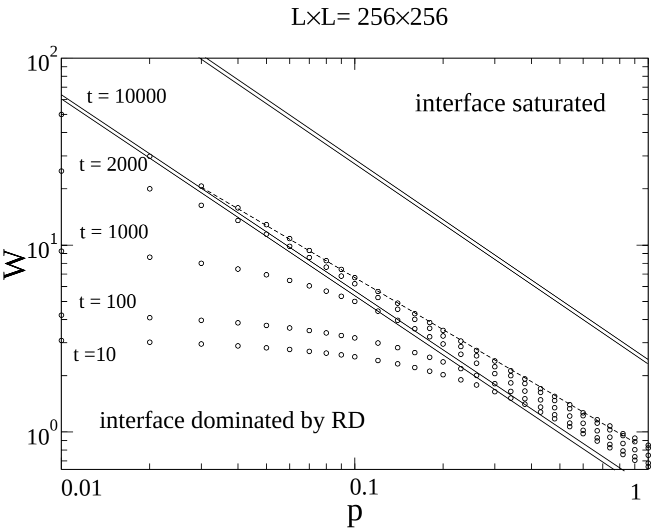

Fig. 3. Log-log plot of W *versus* p for the RDSR/RD model in $d = (2+1)$ dimensions obtained for $L = 256$ and different values of t as indicated in the figure. There are three regions with different behavior as shown in the figure: i) the region dominated by the RD process (left-lower side of the plot), ii) the region corresponding to the saturation of the interface (right-upper side of the plot), and ii) the central region of the plot that corresponds to the intermediate regime. The straight (dashed) line with slope $\gamma = 0.95$ corresponds to the best fit of the data. More details in the text.

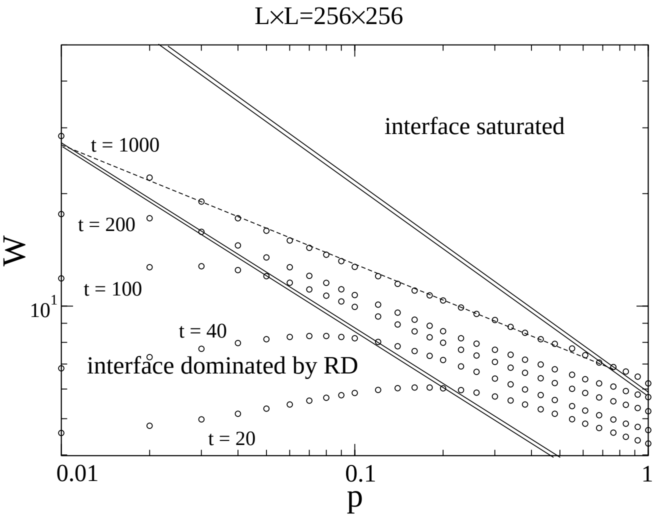

Fig. 4. Log-log plot of W *versus* p for the BD/RD model in $d = (2+1)$ dimensions obtained for $L = 256$ and different values of t as indicated in the figure. The same three regions, exhibiting different behavior, as in the case of RDSR/RD model (Fig. 3), are observed. The straight (dashed) line with slope $\gamma = 0.32$ corresponds to the best fit of the data. More details in the text.

dimensions). Notice that the expected dependence $W \propto \ln(t)$, *i.e.* $\beta = 0$, is shown in the inset of Figure 1b. At a later stage, for $t > t_{x2}$ saturation is observed. The same systematic study as in the case of $d = (1+1)$ dimensions [14] has also been performed. It is found that the interface width $W_s(L, p)$ and the characteristic crossover time t_{x2} also follow equations (4) and (5) where the best fits of the data give $\delta \cong 0.96 \pm 0.04$ and $y \cong 1.9 \pm 0.1$, respectively.

Figure 2a shows log-log plots of W *versus* t obtained for the BD/RD model in $d = (2+1)$ dimensions taking $L = 128$ and different values of p. Also, Figure 2b shows log-log plots of W *versus* t for lattices of different size but keeping $p = 0.16$ constant. Also for this model, as in $d = (1+1)$ dimensions [15], three different regimes are observed. For $t < t_{x1}$, $W(t) \propto t^{\beta_{\mathrm{RD}}}$ with $\beta_{\mathrm{RD}} = 1/2$. During an intermediate time regime ($t_{x1} < t < t_{x2}$), $W(t) \propto t^{\beta_{\mathrm{BD}}}$ with $\beta_{\mathrm{BD}} \sim 0.20$ (this exponent is in agreement with that reported for the BD model in $d = (2+1)$ dimensions [22]). Finally, at a later stage, for $t > t_{x2}$ saturation is observed. As in $d = (1+1)$ dimensions the interface width $W_s(L, p)$ and the characteristic crossover time t_{x2} follow the equations (4) and (5) where the best fits of the data give $\delta \cong 0.45 \pm 0.04$ and $y \cong 0.99 \pm 0.02$, respectively.

Figure 3 (4) shows log-log plots of W *versus* p obtained for the RDSR/RD (BD/RD) model in $d = (2+1)$

dimensions taking $L = 256$ and different values of t. In both cases it is found that equation (6) holds, where for the RDSR/RD model $\gamma \cong 0.95 \pm 0.05$ and for the BD/RD model $\gamma \cong 0.32 \pm 0.02$, respectively.

Simulations of both models, performed in $d = (3+1)$ dimensions exhibit the same behavior as in smaller dimensions (results are not shown here for the sake of space). Furthermore, this behavior can be described by the same exponents. The values of the obtained exponents for both models are listed in Table 1 for $d = (1+1)$, $(2+1)$ and $(3+1)$ dimensions, respectively.

As in $d = (1+1)$ dimensions, the exponents γ, δ and y are not independent. We can generalize the phenomenological dynamic scaling ansatz derived in $d = (1+1)$ dimensions [15] to show that equation (7) is still valid for higher dimensions. In fact, the last column of Table 1 shows that the predictions of equation (7) hold for both models in all the studied dimensions.

It is interesting to note that while the exponents δ and y are independent of the dimensionality, the remaining exponents entering in the relationship given by equation (7), namely β and γ, are dimensionality dependent. So, the latter exponents have to adjust their respective values in order to fit equation (7). This fact becomes nicely apparent in the case of the RDSR/RD model above the critical dimension $d_c = 2$ such as $\beta = 0$ for $d \geq d_c$ implies that γ must remain fixed to $\gamma = 1$. In the case of the BD/RD model no evidence on the existence of a critical dimension is found (as in the case of the KPZ universality class) and consequently both β and γ change with the dimensionality.

Finally, it is possible to conjecture exact values for the exponents in the limit $L \to \infty$, as shown between brackets in Table 1.

4 Phenomenological stochastic growth equations

It is known that the RDSR/RD model in $d = (1+1)$ dimensions can be described by the following phenomenological stochastic growth equation [14]

$$\frac{\partial h(x,t)}{\partial t} = F + \nu_o p^2 \nabla^2 h(x,t) + \eta(x,t), \tag{8}$$

where p is the parameter related to the microscopic dynamics of the model.

In higher dimension we can use scaling arguments to obtain the phenomenological stochastic growth equation. In fact, assuming that the interface is as a function of $\mathbf{x}$ and p it is possible to apply scaling arguments for both coordinates. Furthermore, if the interface $h(\mathbf{x}, p, t)$ is self-similar, then on rescaling it horizontally

$$\mathbf{x} \to \mathbf{x}' \equiv b\mathbf{x}, \tag{9}$$

in the coordinate p

$$p \to p' \equiv cp, \tag{10}$$

and vertically

$$h \to h' \equiv b^{\alpha} c^{-\delta} h, \tag{11}$$

one should obtain an interface that is statistically indistinguishable from the original one. Since the interface roughness depends on time t as well, to compare two interfaces obtained at different times one also must re-scale the time:

$$t \to t' \equiv b^{z} c^{-y} h. \tag{12}$$

Substituting transformations (9–12) into equation (8) in different dimensions and requiring that the resulting equation must be invariant under these transformations we find:

$$z = 2, \quad \alpha = (2-d)/2, \quad \beta = (2-d)/4, \tag{13}$$

and

$$y = 2, \quad \delta = y/2. \tag{14}$$

The relationships given by equation (13) are the well known ones corresponding to the EW universality class [1]. Also, notice that the relations given by equation (14) are independent of the dimensionality. Furthermore, they are in full agreement with both, our previous results obtained in $d = (1+1)$ dimensions [14], and the results presented in this manuscript for $(2+1)$ and $(3+1)$ dimensions (see Tab. 1). So, we conjectured that equation (8) should be valid in all dimensions for the RDSR/RD model.

For the BD/RD one could expect a phenomenological stochastic growth equation similar to the KPZ equation, where the parameters ν and λ should by certain functions depending of p, $\nu(p)$ and $\lambda(p)$, respectively. For the BD/RD model, our previous numerical calculations have shown two main results (Sect. 3): the exponents y and δ are independent of the dimensionality and the relation $\delta = y/2$ is valid (see Tab. 1). Using these findings we can follow similar scaling arguments as in the case of the RDSR/RD model. However, it is already know that this method fails in the case of the KPZ equation, when the coordinate $\mathbf{x}$ is rescaled, because the parameters of this equation (λ and ν) does not re-scale independently [1]. However, it is still possible to use the scaling argument in the coordinate p only, in order to find $\nu(p)$ and $\lambda(p)$.

As already discussed, in the BD/RD model the following phenomenological stochastic growth equation of the KPZ type is expected to hold:

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = F + \nu(p)\nabla^2 h(\mathbf{x},t) + \frac{\lambda(p)}{2}[\nabla h(\mathbf{x},t)]^2 + \eta(\mathbf{x},t). \tag{15}$$

Assuming that the interface $h(\mathbf{x}, p, t)$ is self-similar, then on rescaling it in the coordinate p

$$p \to p' \equiv cp, \tag{16}$$

and vertically

$$h \to h' \equiv c^{-\delta} h, \tag{17}$$

Table 1. List of exponents β, δ, y and γ, measured by means of numerical simulations, for both the RDSR/RD and BD/RD models in different dimensions. The values between brackets are our conjectured exact values for the exponents in the limit $L \to \infty$. The last column shows that equation (7) holds for the measured exponents.

Model	d	β	δ	y	γ	$y\beta - \delta + \gamma$
BD/RD	1+1	$0.30_3(1/3)$	$0.45_1(1/2)$	$0.97_2(1)$	$0.17_1(1/6)$	0.01_3
BD/RD	2+1	0.20_3	$0.45_4(1/2)$	$0.99_2(1)$	0.32_2	0.07_5
BD/RD	3+1	0.06_4	$0.45_4(1/2)$	$0.99_2(1)$	0.40_3	0.01_6
RDSR/RD	1+1	$0.23_3(1/4)$	$0.97_4(1)$	$1.97_5(2)$	$0.51_5(1/2)$	0.01_7
RDSR/RD	2+1	0	$0.96_4(1)$	$1.9_1(2)$	$0.95_5(1)$	-0.01_6
RDSR/RD	3+1	0	$0.95_4(1)$	$1.9_2(2)$	$1.0_1(1)$	0.05_{10}

one should obtain an interface that is statistically indistinguishable from the original one. Since the interface roughness depends on time t as well, one has

$$t \to t' \equiv c^{-y} h. \tag{18}$$

Substituting the scaling transformations (16-18) with $y = 1$ and $\delta = 1/2$ into equation (15) in different dimensions and requiring that the equation must be invariant under these transformations it follows:

$$\nu(p) = \nu p, \tag{19}$$

and

$$\lambda(p) = \lambda p^{3/2}. \tag{20}$$

So, the following equation could be the phenomenological stochastic growth equation for BD/RD model

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = F + \nu p \nabla^2 h(\mathbf{x},t) + \frac{\lambda p^{3/2}}{2}[\nabla h(\mathbf{x},t)]^2 + \eta(\mathbf{x},t). \tag{21}$$

A naive (linear) approach to the development of the growth equations corresponding to both the RDSR/RD and BD/RD models should be to combine the equation of RD (Eq. (1) with $G_i\{h(\mathbf{x},t)\} \equiv 0$) weighted by a factor $(1-p)$, with the EW equation (Eq. (2)) and the KPZ equation (Eq. (3)), weighted by a factor p, respectively. However, as the simulations show this approach is obviously not correct since both equation (8) and equation (21) contain non-linear terms in p.

The physical reasons leading to nonlinear dependences of the coefficients of the growth equations on p is not clear for us. Within this context, it should be noticed that the development of stochastic growth equations describing the crossover between different growth regimes is a subject of ongoing study and that is not fully understood yet [23–27]. For example, according to scaling theories and renormalization group analysis, the crossover time (t_c between the KPZ (with probability p^*) and the EW (with probability $(1-p^*)$) growth regimes is expected to behave as $t_c \propto \lambda^{-\phi} \propto p^{*-\gamma\phi}$, where λ is the usual coefficient of the nonlinear term of the KPZ equation, ϕ and γ are exponents such as $\phi > 0$ since the EW-KPZ crossover vanishes in the limit $\lambda = 0$. Notice that the above relationships imply that $\lambda \propto p^{*\gamma}$. Scaling arguments suggest that $\phi = 4$ in $(1+1)-$dimensions [23], a result that is confirmed by renormalization group calculations [24]. A scaling approach by Amar and Family [28] further support this result. While early numerical results were consistent with $\phi \approx 3$, recent extensive simulations also give $\phi \simeq 4$ and $\gamma = 2.1 \pm 0.02$. So, the nonlinear term of the KPZ equation vanishes almost quadratically ($\lambda \propto p^{*\gamma}$) for $p^* \to 0$ such as only for $p^* = 0$ one has the EW universality class.

On the other hand, there are growth models, such as the Silva-Moreira (SM) model [26], where the parameter λ can be tuned continuously and even change its sign. In fact, combining two models belonging to the KPZ universality class, namely the BD model ($\lambda > 0$) weighted by a factor p^{**} and the restricted SOS Kim-Kosterlitz (KK) model ($\lambda < 0$) weighted by a factor $1 - p^{**}$, one has that the nonlinear term of the resulting model vanishes for $p_c^{**} \simeq 0.83$ [26]. According to the results presented in this paper, combining the SM model (weighted by a factor p) and the RD model (weight factor $1-p$), one should expect a lineal dependence of the type $\nu \propto p$ for $p^{**} \neq p_c^{**}$ (KPZ universality class), while this dependence would switch to a quadratic one for $p^{**} = p_c^{**}$ (EW universality class). So, in spite of the fact that we can not give a plausible physical argument in order to explain the dependence of the coefficients of the growth equations on p, one may expect that the sudden switch from quadratic to linear one, in the limit of vanishing λ, would reflect the change of the universality class that the model undergoes in that limit.

5 Conclusions

Summing up, we have studied in $(1+1)$, $(2+1)$ and $(3+1)$ dimensions the relationships between a microscopic parameter of a growing process and the corresponding stochastic equation. Two models are treated: in the first model, a microscopic parameter p, adjust the probability of the random deposition with relaxation process *versus* the random deposition one. It is found that the stochastic representation for this model is a Edwards Wilkinson equation where the parameter p appears as a factor of the form $\nu(p) = \nu p^2$. In the second model, a microscopic parameter p, adjust the probability of the ballistic deposition process *versus* the random deposition one. In this case an stochastic equation of the KPZ type is found and the parameter p appears in both factors, corresponding to the lineal and non-lineal terms, taking the form $\nu(p) = \nu p$ and $\lambda(p) = \lambda p^{3/2}$, respectively. It is also found that the exponents y and δ are independent of the dimensionality of the substrate. It is shown that the relationship between exponents given by $y\beta - \delta + \gamma = 0$ holds up to $(3 + 1)$ dimensions, and it is conjectured that it may also be valid in higher dimensions.

The derivation of coarse-grained stochastic equations starting from microscopic models is an interesting challenge in the field of Statistical Physics. So, we expect that the relationships between microscopic parameters and stochastic equations presented in this work will stimulate this kind of studies.

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